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Linear Algebra

MAT 226 BA

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## Linear Algebra Fundamentals

### Linear Equations

C1x1+ c2x2 +c3x3+…+cnxn=b

Solve systems of linear equations

Solution set

(s1,s2,s3….)

### Types of solutions

for solving linear equations

1. Unique solution – one solution

Constant – trivial solutions

Independent – trivial solutions

1. Infinite solutions –

At least one free variable

Constant

Dependent

Non-trivial

1. No Solution – inconsistent.

### Ex. Solve the system of linear equations:

Q. How would you solve this?

X1-2x2+x3=0

2x2-8x3=8

-4x1+5x2+9x3=-9

Solution: (29,16,3)

### Coefficient Matrix

A matrix is an Array of numbers

A = [a11,a12…a1n]

[a21,a22….a2n]

[a31,a32….a3n]

Dimension – size of a matrix

Number of rows x by columns

Mxn

So x1 x2 x3

A = eq1 [1, -2, 1]

Eq2 [0, 2, -8]

Eq3 [-4, 5, 9]

### Augmented matrix

Joining of two matrices

A🡨🡪 B

A = eq1 [1, -2, 1, 0]

Eq2 [0, 2, -8, 8]

Eq3 [-4, 5, 9, -9]

How to show if 2 matrixes are equivalent

A = Eq1 [1, -2, 1, 0] [1, 0, 0, a] x1=a

Eq2 [0, 2, -8, 8] ~ [0, 1, 0, b] x2=b

Eq3 [-4, 5, 9, -9] [0, 0, 1, c] x3=c

Unique solution

An example of a consistent system with many infinite solutions

No solution is when the matrix is a number, and the value does not equal those values.

Ex [0,0,0,c] c not =0

Elementary row Operations

1. Interchanging rows

Ri 🡨🡪Rj

1. Multiply a row by a non-zero constant

cRi🡪Ri

1. Add a multiply of a row to another

cRi + Rj 🡪 Rj

Main diagonal of the coefficient matrix

Ex.

Pivot column

Pivot element 🡪

### Upper triangular/ echelon form

To solve a system of linear equations using matrices

Reduce matrices by

If there is a non-zero element in column one(a number in the matrix ) we need to move it to the top row.

Change it to one and make all other entries in column zero

Repeat with each sub matrix

If my pivot element ends up in the last element, I have no solution.

### Echelon form of a matrix

1. All non-zero rows are above any row with all zeros
2. Each leading non-zero entry of a row is to the right of the leading non-zero entry of the row above it.
3. All entries below a leading non-zero entry of a row are zero
   1. For a matrix to be reduced in echelon form the following must also hold:
4. The leading non-Zero entry of each row must be a one
5. All other entries in a column containing the leftmost one must be zeros

## Theorem 1

### Uniqueness of the reduced echelon form

Each matrix is a row equivalent to one and only one reduced echelon form of a matrix

Find the Reduced echelon form of a matrix A:

| 1 4 5 -9 -7 |

System of Linear Equations

Basic Algebraic Solutions within a system

System of Matrix Equations

Matrix algebra solutions within a system

2

Vector spaces and Linear Transformations

Bases and subspaces

Bases of vector spaces

Linear Transformations

Transformations

3

Inner product spaces and spectral theory

Standard inner product spaces derivations

Inner product spaces

Spectral decomposition of a linear transformation

Matrix transformations

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1.3 vectors

Vectors:

W = w1 & W2 are real numbers

Column matrix

Vector w is in